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Fluid Force Activated Spacecraft Dynamics Driven by Gravity Gradient and Jitter Accelerations

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Using as an example the liquid helium dewar of the Gravity Probe-B spacecraft, the dynamics of liquid motions in a slowly rotating, partially liquid fill tank are investigated under the combined action of gravity gradients and *g*-jitter accelerations. The equilibrium configuration of the liquid interface, which is controlled by centripetal acceleration and surface tension, is a torus. Three different cases of liquid responses are examined: 1) when gravity gradient is the dominant driving acceleration, 2) when *g* jitter is the dominant acceleration, and 3) when *g*-jitter and gravity gradient accelerations are comparable. For case 1, the vapor bubble responds in an asymmetric oscillation in which the part of the bubble on one side of the centerline rises while the other side sinks (one-up, one-down oscillation). For case 2, the bubble responds in a combined up-and-down and side-to-side oscillation. The response for case 3 is a combination of the responses for cases 1 and 2. The motion-induced forces and torques are also computed for these cases to quantify the effects on spacecraft control.

Nomenclature

$\hat{a}_{gg}$ = components of gravity gradient vector in cylindrical coordinates, $(a_{gg,r}, a_{gg,\theta}, a_{gg,z})$
$\hat{a}_{gj}$ = components of gravity jitter vector in cylindrical coordinates, $(a_{gj,r}, a_{gj,\theta}, a_{gj,z})$
$\hat{d}$ = vector (not a unit vector) from fluid element to spacecraft mass center
(F_x, F_y, F_z) = fluid force vector exerted on dewar in Cartesian coordinates
f = frequency of gravity jitter, Hz
g = gravity jitter acceleration [Eq. (7)]
g_B = background gravity [Eq. (7)]
g_0 = Earth gravity acceleration, 9.81 m/s^2
L = height of dewar tank, cm
(L_x, L_y, L_z) = fluid moment arm vector exerted on dewar in Cartesian coordinates
(M_x, M_y, M_z) = fluid torque vector exerted on dewar in Cartesian coordinates
N = orbit rate of spacecraft, 1.07×10^{-3} rad/s for GP-B spacecraft
$\hat{n}_\alpha$ = unit vector normal to wall in α direction
p = thermodynamics pressure
(r, θ, z) = axes in cylindrical coordinates
$\hat{r}_c$ = unit vector from spacecraft mass center to center of Earth
t = time
$\hat{t}_\alpha$ = unit vector tangential to wall in α direction
(u, v, w) = velocity components in cylindrical coordinates
(x, y, z) = axes in Cartesian coordinates
$\delta_{\alpha\beta}$ = Kronecker delta
μ = viscous coefficient of fluid
Π = vector of fluid stress exerted on wall
τ = spacecraft gravity turnaround time, s
ψ_E = azimuth angle of Earth toward spacecraft mass center
ω = angular velocity of spacecraft spinning along z axis

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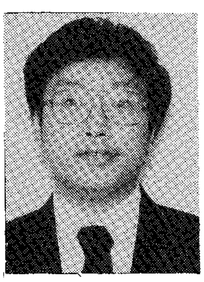

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Subscripts

n	= normal component to wall
t	= tangential component to wall
α, β	= direction of vector

I. Introduction

SOME experimental spacecraft use superconducting sensors for gyroscope read-out and so must be maintained at a very low temperature. The boil-off from the cryogenic liquid used to cool the sensors can also be used, as the Gravity Probe-B (GP-B) spacecraft does, as propellant to maintain attitude control and drag-free operation of the spacecraft.[1] The cryogenic liquid for such spacecraft is, however, susceptible to both sloshlike motion and nonaxisymmetric configurations under the influence of various kinds of gravity jitter and gravity gradient accelerations. Hence, it is important to quantify the magnitude of the liquid-induced perturbations on the spacecraft. Here we use the example of the GP-B to investigate such perturbations by numerical simulations. For this spacecraft disturbances can be imposed on the liquid by atmospheric drag, spacecraft attitude control maneuvers, and Earth's gravity gradient. More generally, onboard machinery vibrations and crew motion can also create disturbances. Recent studies[2–5] suggest that high-frequency disturbances are relatively unimportant in causing liquid motions in comparison to low-frequency ones. The results presented here confirm this conclusion.

After an initial calibration period, the GP-B spacecraft rotates in orbit at 0.1 rpm about the tank symmetry axis. For this rotation rate, the equilibrium liquid-free surface shape is a "doughnut" configuration for all residual gravity levels of $10^{-6}\ g_0$ or less, as shown by experiments[6] and by numerical simulations[3–5]; furthermore, the superfluid behavior of the 1.8 K liquid helium used in GP-B eliminates temperature gradients and therefore such effects as Marangoni convection do not have to be considered. Classical fluid dynamics theory is used as the basis of the numerical simulations here, since Mason's experiments[7] show that the theory is applicable for cryogenic liquid helium in large containers.[8] To study liquid responses to various disturbances, we investigate three levels of gravity jitter (10^{-6}, 10^{-7}, and $10^{-8}\ g_0$) each at three predominant frequencies (0.1, 1.0, and 10 Hz), combined with a gravity gradient appropriate for the GP-B orbit.[9–11]

II. Mathematical Models of Slosh Wave and Fluid: Motion-Induced Stress Fluctuations

For the cases treated here, a full-scale GP-B dewar tank is considered. The tank, which is a cylindrical annulus with ellipsoidal ends, has a major diameter of 136 cm and a height of 145 cm. It is 80% full of cryogenic liquid helium, with the ullage being helium vapor. The tank is rotating about its central axis at 0.1 rpm. In addition, the spacecraft is assumed to turn around in the gravity field once every 600 s (which is coincidentally the same as the spin rate). The contact angle of the liquid on the tank walls is assumed to be 0 deg. The properties of liquid and vapor helium are listed in our previous publications.[5,8,12]

A noninertial (dewar-bound frame) cylindrical coordinate system (r, θ, z) is used for the analysis.[13,14] The corresponding velocity components are (u, v, w), the gravity gradient components are $(a_{gg,r}, a_{gg,\theta}, a_{gg,z})$, and the gravity jitter components are $(a_{gj,r}, a_{gj,\theta}, a_{gj,z})$. The governing continuity and full Navier–Stokes equations in the noninertial frame,[13,14] subject to the initial and boundary conditions[14,15] for this type of problem, are described in our earlier studies.[13–15] In this derivation, Coriolis force, angular acceleration, and centrifugal, viscous, and surface tension forces are given explicitly in the formulations.[13,14]

A staggered velocity grid for the velocity component is used in this computer program. The marker-and-cell method of studying fluid flows along a free surface is adopted.[16–19] The formulation for this method is valid for any arbitrary interface location between the grid points and is not limited to middle-point interfaces.[18,19] There are several methods to be used to solve fluid flow in a free surface. Explicit, semiexplicit, and implicit schemes are the most commonly used methods. Mixing methods for these schemes are employed in this study. A near-semi-implicit PCMI method (predictor–corrector multiple-interaction technique)[20] is used to solve velocity profiles, and Gauss–Seidel,[21] or conjugate gradient,[22] methods are employed to solve pressure iteration, while a successive overrelaxation method[23] is used to compute free-surface configurations through iteration. It has been shown that a near-semi-implicit scheme for the PCMI method is more reliable than that of the explicit scheme and more convenient and saving of computational time than that of the implicit scheme.[24] Second-order accuracy of derivatives are adopted in the finite difference equations whereas the truncation errors for the interior of time and spatial steps are of (Δt^2) and $(\Delta r^2, \Delta\theta^2, \Delta z^2)$, respectively. The convergence criteria for velocity, pressure, and liquid volume are less than 10^{-5}. Some of the steady-state computational results are compared with the experimental observations carried over by Leslie[6] in free-falling aircraft (KC-135) with excellent agreement.

To model the forces and torques exerted on the dewar by large-amplitude liquid motions, the fluid stresses are decomposed into tangential Π_t and normal Π_n components relative to the walls. These expressions are

$$\Pi_t = \mu\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right)\hat{t}_\alpha \hat{n}_\beta \tag{1}$$

$$\Pi_n = p\delta_{\alpha\beta} - \mu\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right)\hat{n}_\alpha \hat{n}_\beta \tag{2}$$

In the computation of how fluid force and torque are fed back to the container, mathematical formulation of the noninertial frame (container-bound coordinate) derived earlier have to be transformed back to inertial frame (Earth-bound coordinate) to carry out the calculation.[14,25] Also, to accommodate the pitching, yawing, and rolling of the spacecraft, it is convenient here to use Cartesian coordinate (x, y, z) with corresponding velocity components. For GP-B, the axis of rotation is always fixed at the proof mass located at the mass center of the dewar $(x_c, y_c, z_c) = (0, 0, \frac{1}{2}L)$. Detailed derivations and expressions for the fluid stress forces (F_x, F_y, F_z) and moments (M_x, M_y, M_z) are given in our recent work[14,25] and so will not be repeated here. The moment arm relating the forces and moments is given by

$$\begin{bmatrix} L_x \\ L_y \\ L_z - \frac{1}{2}L \end{bmatrix} = \frac{1}{F_x^2 + F_y^2 + F_z^2}\begin{bmatrix} F_y M_z - F_z M_y \\ F_z M_x - F_x M_z \\ F_x M_y - F_y M_x \end{bmatrix} \tag{3}$$

The values of gravity acceleration acting on each fluid element in the container at different locations relative to the mass center of the spacecraft, which is orbiting around Earth, can be computed from gravity gradient acceleration.[10,26] Since the interaction between the liquid and the gravity gradient acceleration is capable of exciting liquid motions,[10,13,14,27] this effect is included in the present investigation. The gravity gradient acceleration acting on a fluid particle can be shown to be[10,26,27]

$$\hat{a}_{gg} = -N^2[3(\hat{r}_c\hat{d})\hat{r} - \hat{d}] \tag{4}$$

where, as shown in Fig. 1, $\hat{r}_c$ is the unit vector pointing from the spacecraft mass center to the center of Earth, $\hat{d}$ is the vector from the fluid particle to the spacecraft mass center, and $N = 0.00107$ rad/s is the orbital rate of the spacecraft for the GP-B altitude of 7023 km. It is assumed that the balance of Earth gravity and centrifugal acceleration for the spacecraft is zero at the mass center of the spacecraft. Thus, only the gravity gradient acts on fluid particles, which is different values at different locations in the dewar.[14,27]

The following investigations are aimed in this study: 1) the effect of the motion of a container rotating axis turning along the plane containing two points between the spacecraft mass center and the Earth center, 2) the impact of the gravity gradient and jitter accelerations as the major driving forces affecting the sloshing dynamics of fluid systems, and 3) the correlation between the time-dependent variation of the azimuth angle and the excitation of sloshing waves of fluid systems. Letting ψ_E be the azimuth angle between the spacecraft rotational axis and $\hat{r}_c$, it is assumed that the spacecraft rotational axis is linearly turning around 0–360 deg with respect to $\hat{r}_c$ in $\tau = 600$ s (i.e., at the same rate as the spacecraft spins around the spacecraft

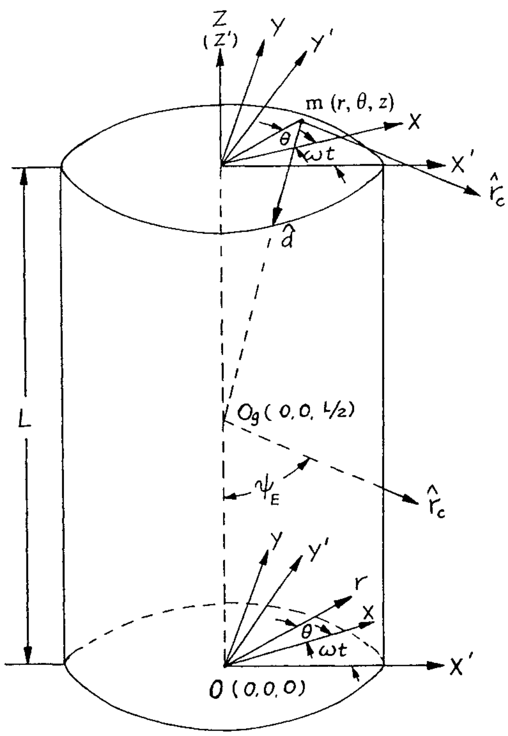


Fig. 1 Coordinate system for computation of gravity gradient acceleration.

axis in this particular case) and that $t = 0$ is the point when the rotational axis is aligned with $\hat{r}_c$. The assumption of $\tau = 600$ s is merely to see the relationship between the rotating axis turn-around period and the spinning period. Thus, the azimuth angle is given by

$$\psi_E = 2\pi t/\tau \tag{5}$$

With these assumptions, the gravity gradient acceleration acting on a fluid particle at (r, θ, z) in the dewar-bound coordinate system is[13,14,27]

$$\hat{a}_{gg} = \begin{bmatrix} a_{gg,r} \\ a_{gg,\theta} \\ a_{gg,z} \end{bmatrix} = -N^2 \begin{bmatrix} \cos(\theta+\omega t) & \sin(\theta+\omega t) & 0 \\ -\sin(\theta+\omega t) & \cos(\theta+\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3\left[\left(z-\frac{1}{2}L\right)\cos\psi_E - r\cos(\theta+\omega t)\sin\psi_E\right]\sin\psi_E + r\cos(\theta+\omega t) \\ r\sin(\theta+\omega t) \\ -3\left[\left(z-\frac{1}{2}L\right)\cos\psi_E - r\cos(\theta+\omega t)\sin\psi_E\right]\cos\psi_E + \left(z-\frac{1}{2}L\right) \end{bmatrix} \tag{6}$$

Fluctuations of the residual gravity (i.e., the g jitter) are modeled by the following equation[9,13]:

$$g = g_B\left[1 + \tfrac{1}{2}\sin(2\pi f t)\right] \tag{7}$$

The components of the jitter in the dewar-bound coordinate system are[9,13,14]

$$\begin{aligned} \hat{a}_{gj} &= (a_{gj,r}, a_{gj,\theta}, a_{gj,z}) \\ &= [g\sin\psi_E\cos(\theta+\omega t), -g\sin\psi_E\sin(\theta+\omega t), -g\cos\psi_E] \end{aligned} \tag{8}$$

Time-dependent effects of the azimuth angle due to gravity turn-around motion for spacecraft in both gravity gradient and jitter accelerations, in addition to Coriolis, angular acceleration, centrifugal, viscous, surface tension forces, etc., have been explicitly included in the mathematical formulation of dewar-bound Navier–Stokes equations[13,14,27] together with boundary conditions.[14,15]

III. Results and Discussion

We will discuss the computed results for various combinations of g-jitter and gravity gradient accelerations. In this study working fluids are liquid and vapor helium II. It is noted that some peculiar behavior of helium fluids with temperature below the λ point (2.17 K) in which helium demonstrates a number of remarkable properties of superfluidity, such as extremely low viscous and surface tension coefficients, reacted to the disturbances driven by gravity gradient and jitter accelerations.[8,28,29] Hung et al.[8] shows that helium bubble disturbances can persist with extremely low damping rate for a long period of time. Also, orbital acceleration, including gravity gradient and jitter accelerations, are basically the time-dependent driving forces.[9,13,14] It can never be expected that the helium bubble disturbances can reach steady state with the combination of peculiar behavior of superfluid helium II and time-dependent orbital accelerations. In this study, time-dependent evolution of first turn-around is computed to study the behavior of the dynamic evolution of bubble disturbances. In this study, gravity gradient acceleration acting on the fluid element inside the GP-B dewar on the orbit is on the order of $10^{-7} g_0$.[13,14,27]

A. Motions Dominated by Gravity Gradient Effects

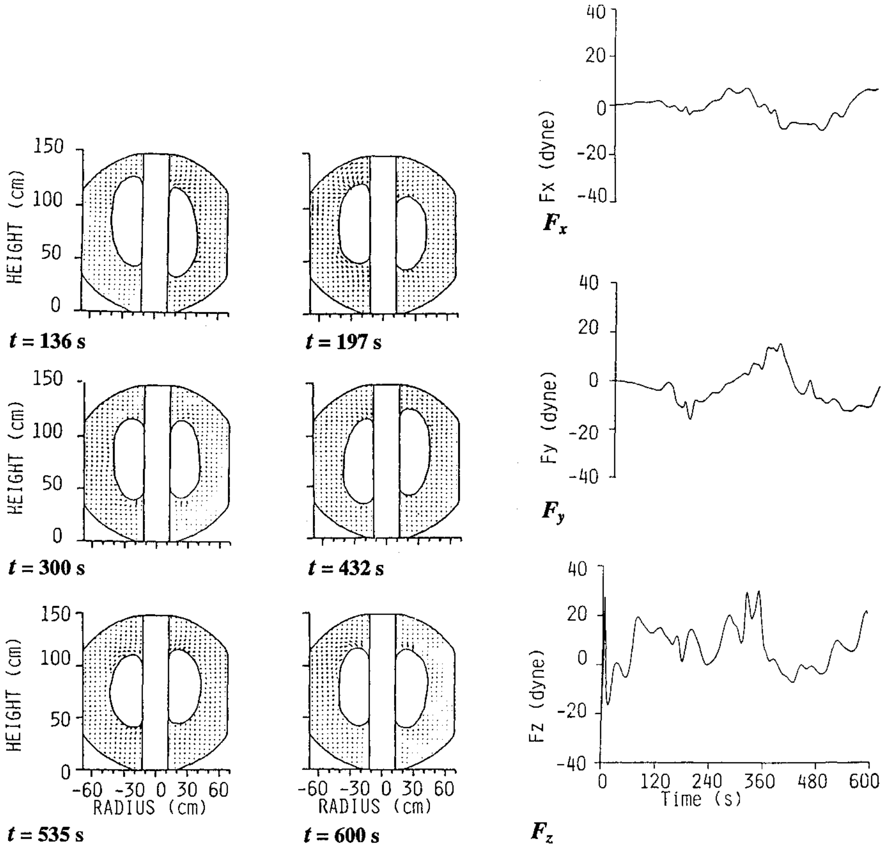
When the magnitude of the g jitter is $10^{-8} g_0$ or less, the liquid interface motions excited by the combined effects of the jitter and gravity gradient accelerations are dominated by the gravity gradient. Figure 2a shows typical results for this case, at illustrative times $t = 136, 197, 300, 342, 535$, and 600 s (which will be used throughout the remainder of the discussion). Equation (6) can be used to show that the gravity gradient acceleration has a larger negative magnitude along the line from the spacecraft center to Earth's center than the positive component transverse to this direction. Figure 2 indicates that this combination of a time-dependent twisting force and torsion moment results in asymmetric bubble deformations in a plane aligned with the gravity gradient acceleration vector. The bubble executes an oscillation in which one side rises when the other side falls, and vice versa. It is expected that this kind of motion will be important for spacecraft control since it exerts a time-dependent moment and twisting force on the spacecraft.[12]

Figure 2b shows the computed liquid forces on the spacecraft. As can be seen, both the fluctuations and the absolute magnitude of the z component of the force are larger than the x and y components because gravity effects are mainly along the axial direction, and the rotational effects of the Coriolis and centrifugal forces equalizing the components are along the x and y directions for dewar-bound coordinates. It should be noted that the initial values of F_x and F_y are both zero at $t = 0$ as a result of symmetry, whereas F_z is not.

Figure 3 shows the computed liquid moments and moment arms for the same case. The magnitudes of the x and y components of the fluctuating moments are nearly equal, as are their absolute magnitudes, and the z component is essentially zero. The time averages of L_x and L_y are nearly zero, in agreement with the near-zero value of M_z. These results reflect the fact that pressure forces created by the liquid motion normal to the tank wall are much more effective in exerting x and y moments than are tangential viscous stresses in creating a z moment (that is, pressure cannot create a net moment around the z axis for an axisymmetric tank).

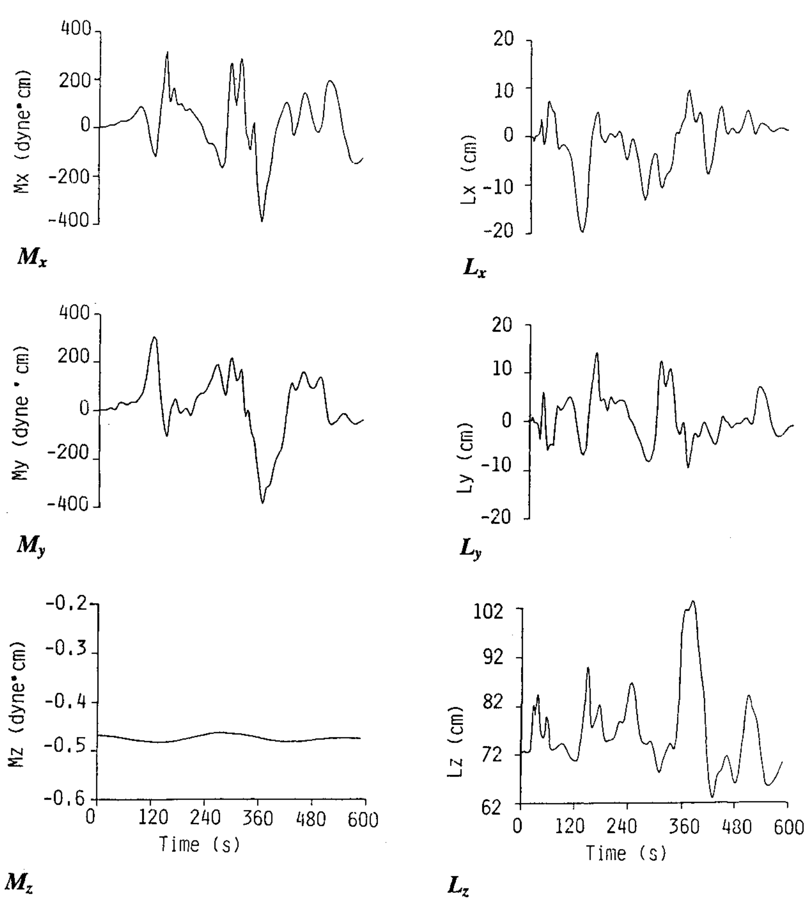
B. Motions Dominated by g-Jitter Effects

When the g-jitter magnitude is $10^{-6} g_0$ or larger, the effects of g jitter dominate over those caused by gravity gradient effects. Figure 4a shows typical results for a jitter frequency of 0.1 Hz for the same illustrative times as used in Fig. 2. It can be shown from Eq. (8) that there is a sinusoidal acceleration in the direction from the spacecraft mass center to Earth's center that combines with the time-dependent variation in the angle ψ_E to produce a vertical and



a) Bubble oscillations b) Fluid forces

Fig. 2 Gravity-gradient-dominated liquid responses ($a_{gj} = 10^{-8}\,g_0$, $f = 0.1$ Hz), time evolution of a) liquid–vapor interface oscillations and b) fluid forces exerted on dewar.



a) Fluid torques b) Fluid moment arms

Fig. 3 Gravity-gradient-dominated liquid responses ($a_{gj} = 10^{-8}\,g_0$, $f = 0.1$ Hz), time evolution of a) pitch, yaw, and roll moments exerted on dewar and b) pitch, yaw, and roll moment arms.

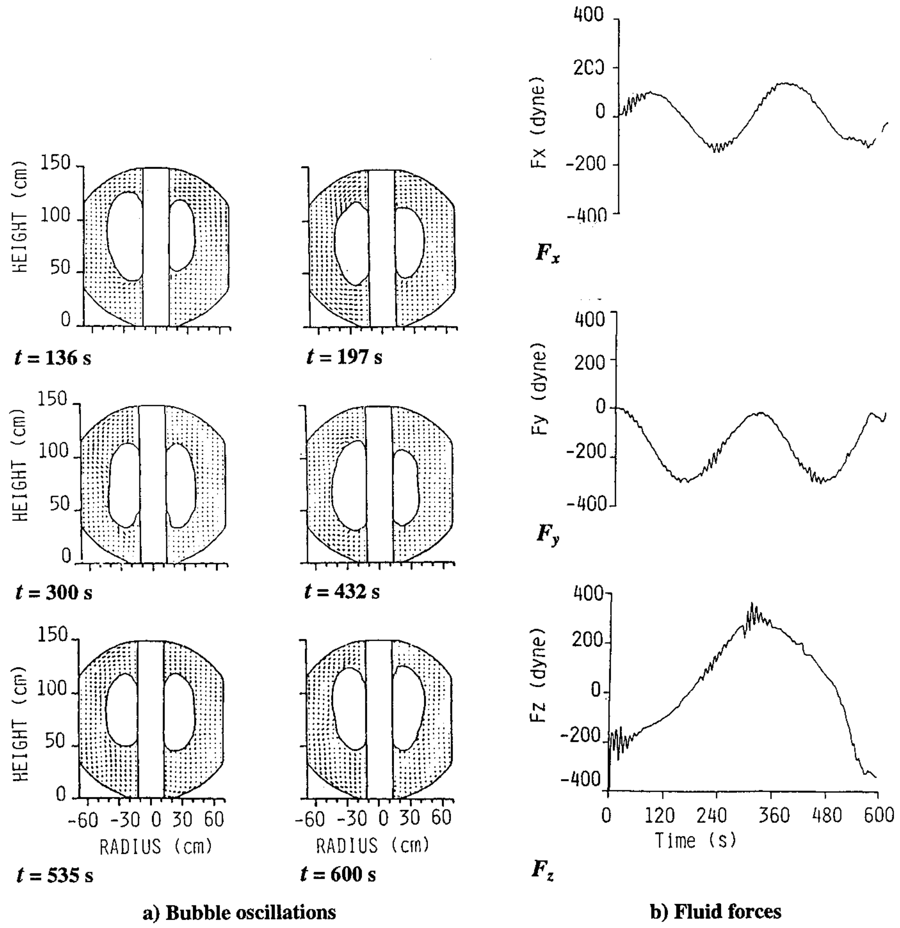


Fig. 4 The g-jitter dominated liquid responses ($a_{gj} = 10^{-6}\,g_0$, $f = 0.1$ Hz), time evolution of a) liquid–vapor interface oscillations and b) fluid forces exerted on dewar.

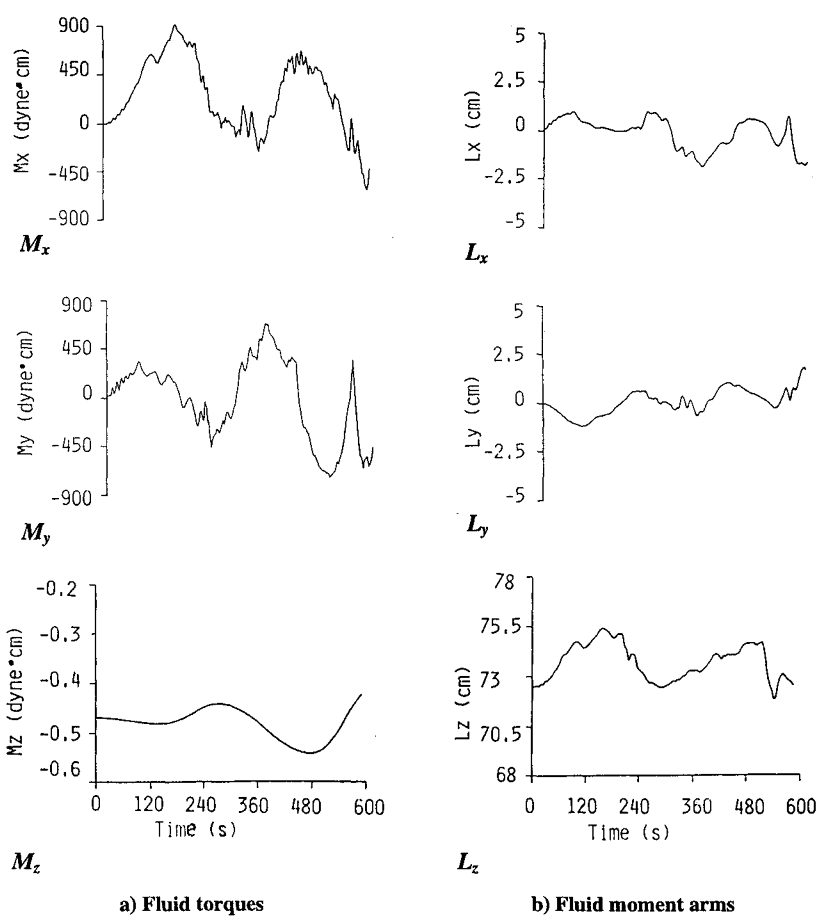
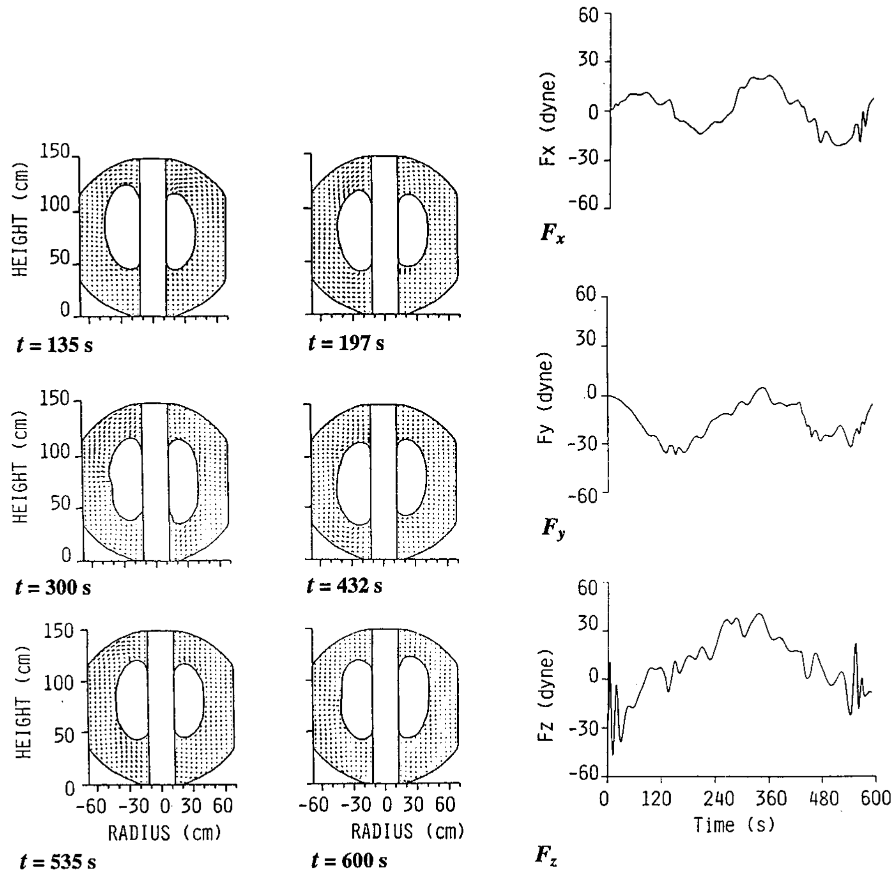
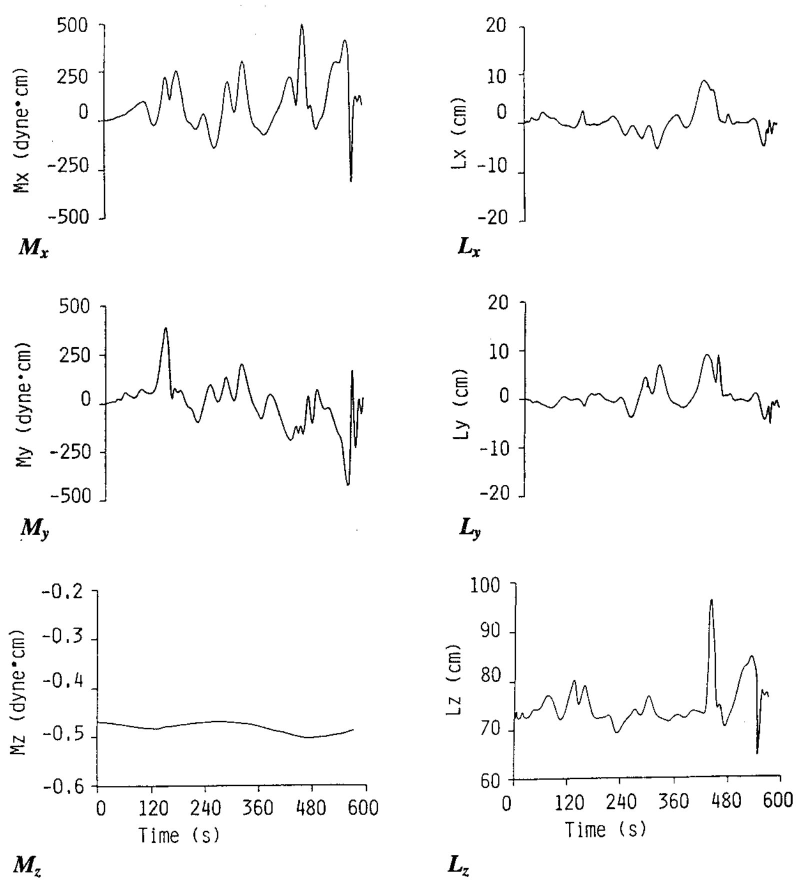


Fig. 5 The g-jitter-dominated liquid responses ($a_{gj} = 10^{-6}\,g_0$, $f = 0.1$ Hz), time evolution of a) pitch, yaw, and roll moments exerted on the dewar and b) pitch, yaw, and roll moment arms.



a) Bubble oscillations b) Fluid forces

Fig. 6 Liquid responses when gravity gradient and g-jitter effects are comparable ($a_{gj} = 10^{-7} g_0$, $f = 0.1$ Hz), time evolutions of a) liquid–vapor interface oscillations and b) fluid forces exerted on dewar.



a) Fluid torques b) Fluid moment arms

Fig. 7 Liquid responses when gravity gradient and g-jitter effects are comparable ($a_{gj} = 10^{-7} g_0$, $f = 0.1$ Hz), time evolution of a) pitch, yaw, and roll moments exerted on dewar and b) pitch, yaw, and roll moment arms.

transverse motion of the bubble. The up-and-down motion is caused primarily by the reversal of the jitter every 300 s. The transverse or sideways motion of the bubble is a result of the coincidence of the periods of the turn-around of the jitter acceleration (i.e., ψ_E variation) and the spacecraft rotation around the z axis; this makes the bubble move between one side of the direction transverse to the axis of rotation to the middle of the axis of rotation.

The simulations for jitter frequencies of 1 and 10 Hz show similar motions but with a much reduced amplitude. This trend with jitter frequency has been well documented in earlier studies[3–5,10,13,14] and so a detailed discussion will not be presented here.

Figure 4b shows the computed forces for this case. The results are qualitatively similar to the gravity gradient dominated case, but the forces have larger magnitudes because the input driving forces are one order of magnitude greater than the gravity gradient dominated case. Figure 5 shows the computed fluid torques and moment arms. Again, the z component of the torque is essentially zero because time average moment arms in x and y components are nearly zero, and the x and y components are nearly equal because of rotational effects of Coriolis and centrifugal forces. The moment and moment arm fluctuations are considerably larger than for the gravity gradient dominated example for the same reasons of greater input driving forces as mentioned earlier.

C. Equal Gravity Gradient and *g*-Jitter Effects

When the g-jitter magnitude is $10^{-7} g_0$, the effects of gravity gradient and g jitter are comparable. Figure 6a shows typical liquid motion results for this case for a jitter frequency of 0.1 Hz. As can be seen, the bubble executes a combined one-side up-and-down, other side down-and-up, sideward and middleward motion that is the sum of the gravity-dominated motion shown in Fig. 2 and the g-jitter-dominated motion shown in Fig. 4.

Figure 6b shows the liquid forces for this case. The trends of the force components are the sum of the two individual effects. Figure 7 shows the computed fluid torques and moment arms. Again, the trends are merely the sum of the two individual effects.

IV. Conclusions

This study of liquid motions in a spinning spacecraft dewar activated by gravity gradient and g jitter has shown that large-amplitude fluctuating fluid forces and torques can be exerted on a spacecraft, particularly along the yaw and pitch axes of fluid torques as a result of such fluid sloshing disturbance induced by these accelerations.

For the purpose of the study, the turn-around time of the gravity field on the spacecraft has been assumed to be the same as the time period for the spacecraft rotation about its symmetry axis (600 s). This combination yields a twisting force and torsion moment on the liquid as well as a sinusoidally oscillating acceleration field. In response, the bubble executes either an up-on-one-side, down-on-the-other-side oscillation driven by the same gradient, or an up-and-down, leftward-and-rightward motion driven by the g jitter, or a combination motion when the gravity gradient and g jitter have comparable magnitudes. The liquid motions create large-amplitude fluctuations in fluid force and torques on the tank that can impose yaw and pitch control problems. The asymmetric orientation of the liquid in the tank as a result of the gravity gradient may also create control and pointing problems.

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